then

$$a_{11} = k^2K_{33}/K_{11}$$
$$a_{12} = -\omega\mu_0\gamma K_{12}/K_{11}$$
$$a_{21} = \omega\epsilon_0\gamma K_{12}K_{33}/K_{11}$$
$$a_{22} = k^2 + k_1^2K_{12}/K_{11}.$$

The currents $J_1$ and $J_2$ are divided into two parts, one part arising from the longitudinal currents and the other part from the transverse currents

$$J_1 = J_{1z} + J_{1t}$$

where

$$J_{1z} = \frac{j}{\omega}\left[\frac{k^2}{\epsilon_0 K_{11}} J_{ze} - \frac{a_{12}}{\mu_0} J_{zm}\right]$$

and

$$J_{1t} = \nabla_t \cdot \left[\bar{u}_z \times \bar{J}_{tm} - j\frac{\gamma}{\omega\epsilon_0 K_{11}}\bar{J}_{te}\right].$$

Similarly

$$J_2 = J_{2z} + J_{2t}$$

where

$$J_{2z} = -\frac{j}{\omega\mu_0}\left[a_{22}J_{zm} + a_{12}J_{ze}\right]$$

and

$$J_{2t} = \nabla_t \cdot \left[\bar{u}_z \times \bar{J}_{te} - j\frac{K_{12}}{K_{11}}\bar{J}_{te} + j\frac{\gamma}{\omega\mu_0}\bar{J}_{tm}\right].$$

The transverse fields can be expressed in terms of the longitudinal fields and transverse currents as follows:

$$\begin{aligned}\bar{E}_t = \frac{1}{(k^4 - k_1^4)}\{&\nabla_t(-\gamma k^2 E_z - \omega\mu_0 k_1^2 H_z)\\ &+ j\bar{u}_z \times \nabla_t(\gamma k_1^2 E_z + \omega\mu_0 k^2 H_z)\\ &+ \bar{u}_z \times (\gamma k^2 \bar{J}_{tm} + \omega\mu_0 k_1^2 \bar{J}_{te})\\ &+ j(\omega\mu_0 k^2 \bar{J}_{te} + \gamma k_1^2 \bar{J}_{tm})\}\end{aligned} \qquad (2a)$$

$$\begin{aligned}\bar{H}_t = \frac{1}{(k^4 - k_1^4)}\{&\nabla_t(-\gamma k^2 H_z + \gamma^2\omega\epsilon_0 K_{12} E_z)\\ &+ j\bar{u}_z \times \nabla_t[\gamma k_1^2 H_z - \omega\epsilon_0(k_1^2 K_{12}\\ &+ k^2 K_{11})E_z]\\ &+ \bar{u}_z \times (\gamma k^2 \bar{J}_{te} - \omega\epsilon_0\gamma^2 K_{12}\bar{J}_{tm})\\ &+ j[k_1^2\gamma\bar{J}_{te} - \omega\epsilon_0(k_1^2 K_{12}\\ &+ k^2 K_{11})\bar{J}_{tm}]\}.\end{aligned} \qquad (2b)$$

If $a_{12} = a_{21} = 0$ then (1) represents two uncoupled equations which can be solved for $E_z$ and $H_z$; these expressions for $E_z$ and $H_z$ can be used in (2a) and (2b) to determine the transverse fields. The matrix elements $a_{12}$ and $a_{21}$ are both zero if $K_{12} = 0$ or $\gamma = 0$.

If $a_{12}$ and $a_{21}$ are not both zero, then (1) can be uncoupled by a suitable linear transformation of the form

$$F = TU \qquad (3)$$

where $T$ is a $2 \times 2$ matrix and $U$ is a two-dimensional vector.

Substituting (3) into (1) and premultiplying by $T^{-1}$ gives

$$\nabla_t^2 T^{-1}TU + T^{-1}ATU = T^{-1}J.$$

Using the theory of similarity transformations, it is possible to find a matrix $T$ which will diagonalize the matrix $A$, giving

$$T^{-1}AT = D(p_1^2, p_2^2)$$

where $p_1^2$ and $p_2^2$ are two solutions of the characteristic equation of $A$

$$\det (A - p^2 I) = 0. \qquad (4)$$

Making such a transformation (1) becomes

$$\nabla_t^2 U + D(p_1^2, p_2^2)U = T^{-1}J. \qquad (5)$$

Solutions of (4) are available in the literature in terms of the elements of $\overline{\overline{K}}$ [2].

If neither $a_{12}$ nor $a_{21}$ are zero, and if $p_1^2 \neq p_2^2$, then an appropriate matrix $T$, giving the same transformation used by Kales [3] in a similar derivation for a source free ferrite, is

$$T = \begin{bmatrix} \dfrac{(p_1^2 - a_{22})p_1^2}{a_{21}} & \dfrac{(p_2^2 - a_{22})p_2^2}{a_{21}} \\ p_1^2 & p_2^2 \end{bmatrix}$$

the inverse of which is

$$T^{-1} = \begin{bmatrix} \dfrac{a_{21}}{p_1^2(p_1^2 - p_2^2)} & \dfrac{(p_2^2 - a_{22})}{p_1^2(p_2^2 - p_1^2)} \\ \dfrac{a_{21}}{p_2^2(p_2^2 - p_1^2)} & \dfrac{p_1^2 - a_{22}}{p_2^2(p_1^2 - p_2^2)} \end{bmatrix}.$$

The longitudinal fields are now obtainable from the solutions of (5) through the use of (3). The transverse field expressions written in terms of the solutions of (5) and the transverse currents are

$$\begin{aligned}\bar{E}_t = &\frac{1}{\omega\epsilon_0 K_{12}}\nabla_t[(k^2 - p_1^2)U_1 + (k^2 - p_2^2)U_2]\\ &+ j\omega\mu_0\bar{u}_z \times \nabla_t(U_1 + U_2)\\ &+ \frac{1}{k^4 - k_1^4}\{\bar{u}_z \times (\gamma k^2\bar{J}_{tm} + \omega\mu_0 k_1^2\bar{J}_{te})\\ &+ j(\omega\mu_0 k^2\bar{J}_{te} + \gamma k_1^2\bar{J}_{tm})\}\end{aligned}$$

$$\begin{aligned}\bar{H}_t = &-\gamma\nabla_t(U_1 + U_2) - j\frac{K_{11}}{\gamma K_{12}}\bar{u}_z\\ &\times \nabla_t[(p_1^2 - a_{22})U_1 + (p_2^2 - a_{22})U_2]\\ &+ \frac{1}{k^4 - k_1^4}\{\bar{u}_z \times (\gamma k^2\bar{J}_{te} - \omega\epsilon_0\gamma^2 K_{12}\bar{J}_{tm})\\ &+ j[k_1^2\gamma\bar{J}_{te} - \omega\epsilon_0(k_1^2 K_{12} + k^2 K_{11})\bar{J}_{tm}]\}.\end{aligned}$$

Making a transformation from a gyroelectric medium to a gyromagnetic medium the equations presented here reduce to those obtained by Rosenbaum and Coleman [4] for a ferrite containing longitudinal electric currents only.

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# On the Superheterodyne Method of Microwave Noise Measurements

One of the frequently used methods in microwave oscillator noise measurements is the superheterodyne method. This correspondence is not intended to describe this method in detail, but to call attention to the possibility of pointing out one measurement error. Every one who is interested in microwave oscillator measurements can find fundamental information given in [1].

The superheterodyne method is assumed to be a substitution method, and therefore it is not necessary to know the mixer crystal noise [1], [2]. It will be shown that some conditions have to be fulfilled if the superheterodyne method is to become a substitution method which gives correct values of the oscillator noise power.

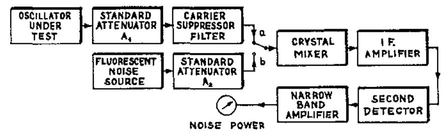


Fig. 1. Schematic diagram of the superheterodyne method noise measuring circuit.

The schematic diagram of the noise measuring equipment is given in Fig. 1. When the oscillator under test is connected to the input of the superheterodyne receiver [case (a)], it produces a reading on the output meter

$$N_1 = \left(\frac{N_o}{L_c A_1} + N_{c1} + N_a\right) G_a \qquad (1)^{1,2}$$

where

$N_o$ = noise power of the oscillator under test
$N_{c1}$ = noise power contributed by the mixer crystals for the case (a)
$N_a$ = noise power contributed by the IF amplifier
$G_a$ = amplifier gain
$L_c$ = conversion loss of mixer crystals
$A_1$ = attenuation factor of the standard attenuator in the oscillator arm.

When the tested oscillator is removed and a known amount of noise from the noise source is added [case (b)] the reading is

$$N_2 = \left(\frac{N_n}{L_c A_2} + N_{c2} + N_a\right) G_a \qquad (2)$$

where

$N_n$ = output noise power of the noise source
$N_{c2}$ = noise power contributed by the mixer crystals for case (b)
$A_2$ = attenuation factor of the standard attenuator in the noise source arm.

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[1] The exact form of (1) and others similar in this correspondence is

$$N = \left(\frac{N_o - kT_o}{L_c A_1} + N_c + N_a\right) G_a.$$

However, usually $N_o \gg kT_0$.

[2] All noise powers $N$ is this correspondence are considered as noise powers in unity bandwidth.

Thus because $N_1 = N_2$,

$$N_o = \frac{N_n A_1}{A_2} + A_1 L_c (N_{c2} - N_{c1}). \quad (3)$$

It is known that the noise powers contributed by the mixer crystals in both cases are not always identical [3], [4]. In case (a), one can derive an expression for this noise component, following the considerations in [4]–[6] as

$$N_{c1} = \left[ t + \left( \frac{\beta P_{LO}}{f_m^{\gamma}} + 1 \right) \frac{\alpha^2 R_{IF} P_o}{L_c A_1 S_f} \right] k T_0 \quad (4)^3$$

where

$$t = \frac{\beta P_{LO}}{f_{IF}^{\gamma}} + 1$$

$t$ = temperature noise ratio of the crystals on the frequency $f_{IF}$
$\beta_1 \gamma$ = constants characterizing the mixer crystals
$\alpha$ = nonlinearity coefficient of the crystals
$R_{IF}$ = IF impedance of the mixer crystals
$P_{LO}$ = local oscillator power level at the mixer crystals
$P_o$ = output carrier power level of the oscillator under test
$S_f$ = suppression factor of the carrier suppression filter
$f_{IF}$ = frequency of the IF amplifier
$f_m$ = mean frequency of the narrow-band amplifier connected next to the second detector
$k$ = Boltzmann's constant
$T_0$ = temperature in degrees Kelvin.

In case (b) the mixer crystals noise contribution is

$$N_{c2} = t k T_0. \quad (5)$$

Substituting (4) and (5) into (3), one can obtain the noise power of the oscillator under test

$$N_o = \frac{N_n A_1}{A_2} - \left( \frac{\beta P_{LO}}{f_m^{\gamma}} + 1 \right) \frac{\alpha^2 R_{IF} P_o}{S_f} k T_0. \quad (6)$$

After analyzing this final result it becomes clear that it is impossible, in the general case, to eliminate the influence of the mixer crystals noise in the superheterodyne method. The mixer crystal noise is allowed for only in the case when

$$N_{c1} = N_{c2}. \quad (7)$$

Equation (6) shows that if the carrier power level of the oscillator under test at the input of the mixer is kept small,[4] or if the temperature noise ratio of the crystals used in the superheterodyne mixer is small enough (Doppler crystals), the condition (7) will be fulfilled. Only then will (6) have the following form:

$$N_o = \frac{N_n A_1}{A_2}. \quad (8)$$

This equation was assumed heretofore as always valid by all investigators, as far as it is known to the author.

One can give an expression for the measurement error which is made when (8) is always assumed valid:

$$\delta = \frac{\left( \frac{\beta P_{LO}}{f_m^{\gamma}} + 1 \right) \frac{\alpha^2 R_{IF} P_o}{S_f} k T_0}{\frac{N_n A_1}{A_2} - \left( \frac{\beta P_{LO}}{f_m^{\gamma}} + 1 \right) \frac{\alpha^2 R_{IF} P_o}{S_f} k T_0}. \quad (9)$$

One can estimate the value of this error in a simple example. Suppose a reflex klystron with carrier power level $P_o = 30$ mW has been measured. The noise power in unity bandwidth of the fluorescent noise source is $N_n = 1.42 \cdot 10^{-19}$ W/Hz (15.5 dB). The equal crystals used in the superheterodyne receiver have parameters

$$\alpha = 20\ V^{-1}$$

$$t_m = \left( \frac{\beta P_{LO}}{f_m^{\sigma}} + 1 \right)$$

$$= 10^4\ (40\ \text{dB}) - \text{temperature noise ratio}$$

$$R_{IF} = 200\Omega.$$

The used carrier suppression filter has a suppression factor $S_f = 2.10^3$ (~33 dB). The two typical measurements have given the following results:

case (a): $A_1 = 500$ (~27 dB)
case (b): $A_2 = 1$ (0 dB).

Computing the carrier/noise factor of the measured klystron on the bases of (8), one obtains the following result: $P_o/N_o = 146.3$ dB/Hz. On the other hand, based on the exact equation (6), one obtains $P_o/N_o = 151.15$ dB/Hz. The measurement error determined on the bases of (9) is $\delta = 209$ per cent.

The foregoing example shows that the only condition given so far, that the tested oscillator carrier power level should be at least 10 dB below that of the local oscillator [1], is unsufficient, in practical cases, to accomplish (7).

The determination of sufficient conditions for correctness of the superheterodyne method is, in general, a complicated problem. In practice, however, one can confine oneself to determining the admissible carrier power level $P_o' = P_o / A_1 S_f$ for the used superheterodyne receiver. Below that level (8) is valid. This can be done experimentally, by determining the noise power $N_o$ on the bases of (8), for several values of $P_o'$ changing the suppression factor $S_f$. [Changes of $A_1$ have no influence on measurement results; see (6).] For power levels smaller than the admissible level, $N_o$ given by (8) will be constant. Only then will the measurement error be permissibly small and the results of oscillator noise correct; and then the superheterodyne method may be thought of as a substitution method.

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[3] Expression (4) is also true for the case of a ESR spectrometer with superheterodyne detection. Then $f_m$ is the modulation frequency of the magnetic field.
[4] By using an effective carrier suppression filter.

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